

Reinforcement Learning: Algorithms and Applications

Learning from Interaction

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What is Reinforcement Learning?

- Learning through interaction with an environment
- No explicit supervision - learning from rewards and punishments
- Goal: Learn optimal behavior to maximize cumulative reward
- Inspired by behavioral psychology and animal learning
- Different from supervised and unsupervised learning

Key Characteristics

- **Trial-and-error learning:** Agent explores different actions
- **Delayed consequences:** Actions may have long-term effects
- **Exploration vs Exploitation:** Balance between trying new actions and using known good ones
- **Sequential decision making:** Decisions affect future states
- **No labeled examples:** Learning from scalar reward signals

The Reinforcement Learning Framework

- **Agent:** The learner/decision maker
- **Environment:** Everything the agent interacts with
- **State (S):** Current situation/configuration
- **Action (A):** What the agent can do
- **Reward (R):** Immediate feedback from environment
- **Policy (π):** Strategy for choosing actions

The Agent-Environment Interaction

At each time step t :

- ① Agent observes state S_t
- ② Agent selects action A_t based on policy π
- ③ Environment responds with:
 - Next state S_{t+1}
 - Reward R_{t+1}
- ④ Process repeats...

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \dots$$

Markov Decision Process (MDP)

An MDP is defined by:

- S : Set of states
- A : Set of actions
- P : Transition probabilities $P(s'|s, a)$
- R : Reward function $R(s, a, s')$
- γ : Discount factor $[0, 1]$

Markov Property: Future depends only on current state, not history

$$P(S_{t+1} = s' | S_t = s, A_t = a, S_{t-1}, A_{t-1}, \dots) = P(S_{t+1} = s' | S_t = s, A_t = a)$$

Return and Value Functions

Return: Total discounted reward from time t

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

State Value Function: Expected return starting from state s

$$V^{\pi}(s) = E_{\pi}[G_t | S_t = s]$$

Action Value Function: Expected return from state s , action a

$$Q^{\pi}(s, a) = E_{\pi}[G_t | S_t = s, A_t = a]$$

Bellman Equations

Bellman Equation for State Values:

$$V^\pi(s) = \sum_a \pi(a|s) \sum_{s'} P(s'|s, a) [R(s, a, s') + \gamma V^\pi(s')]$$

Bellman Equation for Action Values:

$$Q^\pi(s, a) = \sum_{s'} P(s'|s, a) [R(s, a, s') + \gamma \sum_{a'} \pi(a'|s') Q^\pi(s', a')]$$

Optimal Bellman Equations:

$$V^*(s) = \max_a \sum_{s'} P(s'|s, a) [R(s, a, s') + \gamma V^*(s')]$$

Policy-based vs Value-based Methods

Value-based Methods

- Learn value functions
- Derive policy from values
- Examples: Q-learning, SARSA
- Good for discrete actions

Policy-based Methods

- Directly learn policy
- Parameterized policies
- Examples: REINFORCE, Actor-Critic
- Handle continuous actions well

Actor-Critic Methods: Combine both approaches

- Actor: Policy component
- Critic: Value function component

Q-Learning: Off-Policy Temporal Difference

Key Idea: Learn optimal action values $Q^*(s, a)$ directly

Update Rule:

$$Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$$

Where:

- α : Learning rate
- r : Immediate reward
- γ : Discount factor
- $\max_{a'} Q(s', a')$: Maximum Q-value in next state

Policy: $\pi(s) = \arg \max_a Q(s, a)$ (greedy)

Q-Learning Algorithm

- ❶ Initialize $Q(s, a)$ arbitrarily for all s, a
- ❷ For each episode:
 - ❶ Initialize state s
 - ❷ For each step of episode:
 - ❶ Choose action a using policy derived from Q (e.g., ϵ -greedy)
 - ❷ Take action a , observe reward r and next state s'
 - ❸ Update: $Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$
 - ❹ $s \leftarrow s'$
 - ❸ Until s is terminal

Exploration Strategies

ϵ -greedy:

- With probability ϵ : choose random action
- With probability $1 - \epsilon$: choose $\arg \max_a Q(s, a)$

Softmax/Boltzmann:

$$P(a|s) = \frac{e^{Q(s,a)/\tau}}{\sum_{a'} e^{Q(s,a')/\tau}}$$

Upper Confidence Bound (UCB):

$$a_t = \arg \max_a \left[Q(s, a) + c \sqrt{\frac{\ln t}{N(s, a)}} \right]$$

Policy Gradient Approach

Parameterized Policy: $\pi_{\theta}(a|s)$

Objective: Maximize expected return

$$J(\theta) = E_{\pi_{\theta}}[G_t]$$

Policy Gradient Theorem:

$$\nabla J(\theta) \propto \sum_s d^{\pi}(s) \sum_a Q^{\pi}(s, a) \nabla \pi_{\theta}(a|s)$$

REINFORCE Update:

$$\theta \leftarrow \theta + \alpha G_t \frac{\nabla \pi_{\theta}(A_t|S_t)}{\pi_{\theta}(A_t|S_t)}$$

Actor-Critic Methods

Combines:

- Policy gradient (Actor)
- Value function approximation (Critic)

Actor Update:

$$\theta \leftarrow \theta + \alpha \delta \frac{\nabla \pi_{\theta}(A_t|S_t)}{\pi_{\theta}(A_t|S_t)}$$

Critic Update:

$$w \leftarrow w + \beta \delta \nabla V_w(S_t)$$

Where $\delta = R_{t+1} + \gamma V_w(S_{t+1}) - V_w(S_t)$ is the TD error

- **Game Playing:** Chess, Go, Atari games, StarCraft II
- **Robotics:** Robot navigation, manipulation, walking
- **Autonomous Systems:** Self-driving cars, drones
- **Finance:** Algorithmic trading, portfolio management
- **Healthcare:** Treatment recommendations, drug discovery
- **Resource Management:** Traffic control, power grid optimization
- **Natural Language:** Dialogue systems, machine translation
- **Recommendation Systems:** Content recommendation, advertising

- **AlphaGo/AlphaZero**: Mastered Go, Chess, and Shogi
- **DQN**: Human-level performance on Atari games
- **OpenAI Five**: Competed in Dota 2 tournaments
- **AlphaStar**: Achieved Grandmaster level in StarCraft II
- **GPT/ChatGPT**: Large language models with RL fine-tuning
- **Autonomous Vehicles**: Tesla, Waymo self-driving systems
- **Data Center Cooling**: Google's 40% energy reduction

Current Challenges

- **Sample Efficiency:** Need many interactions to learn
- **Exploration:** Finding good strategies in large state spaces
- **Generalization:** Transferring knowledge to new environments
- **Partial Observability:** Dealing with incomplete information
- **Multi-Agent Settings:** Learning with other agents
- **Safety:** Ensuring safe exploration and deployment
- **Interpretability:** Understanding learned policies
- **Reward Engineering:** Designing appropriate reward functions

Advanced Topics and Extensions

- **Deep Reinforcement Learning:** Neural networks as function approximators
- **Multi-Agent RL:** Learning in multi-agent environments
- **Hierarchical RL:** Learning at multiple temporal abstractions
- **Transfer Learning:** Applying knowledge across domains
- **Imitation Learning:** Learning from expert demonstrations
- **Safe RL:** Incorporating safety constraints
- **Meta-Learning:** Learning to learn quickly
- **Offline RL:** Learning from fixed datasets

- **More Sample-Efficient Algorithms**
- **Better Exploration Strategies**
- **Robust and Safe RL Systems**
- **Integration with Other ML Paradigms**
- **Real-World Deployment Challenges**
- **Ethical Considerations and Fairness**
- **Quantum Reinforcement Learning**
- **Continual and Lifelong Learning**

Key Takeaways

- RL enables learning optimal behavior through interaction
- Balancing exploration and exploitation is crucial
- Value-based and policy-based methods offer different advantages
- Deep RL has achieved remarkable successes in complex domains
- Many challenges remain for real-world deployment
- Active area of research with promising future applications

Questions?

"The only way to make sense out of change is to plunge into it, move with it, and join the dance."

- Alan Watts

(This quote reflects the essence of reinforcement learning - learning through interaction and adaptation)